Calculus of Communicating Systems

Tuesday, September 06, 2011
13:23

Last Time

→ CCS

→ Structural Congruence

Today

→ More CCS

→ Examples

→ Define a LTS for CCS

→ Some theorems (metatheory)

Announcements

→ Office Hours: 11-12 Wed 4137 Upson

→ Project Proposals due Friday 9/16 5PM

→ \jnf false in Note template is for prettyness.

Note: CCS is basically π-Calculus w/ only data less channels.

set of all labels

disjoint union

$\mathcal{L} = N \cup \overline{N}$

names

complementary names.

instances of $\mathcal{L}$

$\lambda, \mu, \ldots$

instances of Act

$Act = \mathcal{L} \cup \{\tau\}$

$\alpha, \beta, \ldots$

any capital Roman letter

$P_1 := A\langle a_1, \ldots, a_n \rangle \mid \sum_{i \in I} a_i P_i \mid P_1 | P_2 \mid \text{new } a\, P$

$A(a_1, \ldots, a_n) = P_A$

The most important rule is the "react" rule:

$$\frac{}{(a.P + M) \mid (\overline{a}.Q + N) \to P | Q} \text{ React Rule}$$

this is a Σ

This is an unrolled Σ, unrolled by 1 term

$$\frac{P \to P'}{P | Q \to P' | Q} \text{ Parallel Rule}$$

$$\frac{P \to P'}{\text{new } a\, P \to \text{new } a\, P'} \text{ Restriction Rule}$$

Also

→ See structural congruence from last time

last time.

$$\frac{}{\tau . P + M \to P} \quad \text{Tau Rule}$$

internal transitions

structural congruence

$$\frac{Q \equiv P \to P' \equiv Q'}{Q \to Q'}$$

<u>Example</u>

Suppose we want to model a lottery, the kind where you choose a ball, and then we can choose a new ball

decide to pick a ball

$$Lottery = \tau . b_1 . Lottery + \dots + \tau . b_n . Lottery$$

choose a ball

recurse to choose a new ball

note that this doesn't allow a multi-ball lottery.

$$(Lottery \mid \overline{b_1} . P_1 \mid \dots \mid \overline{b_n} . P_n)$$

if b_1 was picked, P_1 is a winner!

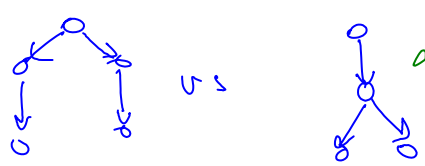
one ball lottery

<u>Multi Ball Lottery</u>

$$(Lottery \mid \overline{b_1} . \overline{b_2} . P_{12} \mid \overline{b_1} . \overline{b_3} . P_{13} \mid \dots)$$

causes deadlock. Need basic idea as

$$\overline{b_1} . (\overline{b_2} . P_{12} \mid \overline{b_3} . P_{13})$$



$$A(a,b,c) = \overline{a} . C\langle a,b,c \rangle$$

$$B(a,b,c) = b . C\langle a,b,c \rangle$$

$$C(a,b,c) = \tau . B\langle a,b,c \rangle + c . A\langle a,b,c \rangle$$

$C_i \Rightarrow b_i$ ball <u>may</u> be picked

$$A_i = A\langle a_i, b_i, a_{i+1} \rangle$$

$$B_i = B\langle a_i, b_i, a_{i+1} \rangle$$

kinda like #define's, to be used ...

$B_i = B\langle a_i, b_i, a_{i+1}\rangle$

$C_i = C\langle a_i, b_i, a_{i+1}\rangle$

be used w/

indices are $(\# \bmod 3) + 1$

<u>Back to the lottery:</u>

$L_1 = \text{new } a_1, a_2, a_3\ (C_1 \mid A_2 \mid A_3)$

either pick a ball or switch states to maybe pick a different ball

$\equiv \text{new } a_1, a_2, a_3\ (\tau \cdot B\langle a_1, b_1, a_2\rangle + a_2 \cdot A\langle a_1, b_1, a_2\rangle$

$\mid \overline{a_2} \cdot C\langle a_2, b_2, a_3\rangle$

$\mid \overline{a_3} \cdot C\langle a_3, b_3, a_1\rangle$

L_1

$\text{new } a_1, a_2, a_3\ (A\langle a_1, b_1, a_2\rangle \mid C\langle a_2, b_2, a_3\rangle \mid A_3)$

$\equiv \text{new } a_1, a_2, a_3\ (A_1 \mid C_2 \mid A_3)$

$\text{new } a_1, a_2, a_3\ (b_1 \cdot C\langle a_1, b_1, a_2\rangle \mid A_2 \mid A_3)$

$\downarrow$

$\text{new } a_1, a_2, a_3\ (C_1 \mid A_2 \mid A_3)$

$L_1 = \text{new } \vec{a}\ (C_1 \mid A_2 \mid A_3)$ pick b_1

$\downarrow$ τ transition

$L_2 = \text{new } \vec{a}\ (A_1 \mid C_2 \mid A_3)$ pick b_2

$\downarrow$ τ transition

$L_3 = \text{new } \vec{a}\ (A_1 \mid A_2 \mid C_3)$ pick b_3

<u>Now</u>, let's build a LTS!

$\longrightarrow$ This lets us differentiate τ from non-τ transition

L-sum

$\mu + \alpha \cdot P + N \xrightarrow{\alpha} P$

<u>Notes</u> $\rightarrow$ w/o

label refers to earlier system of CCS described earlier

$$\frac{P \xrightarrow{\lambda} P' \qquad Q \xrightarrow{\bar{\lambda}} Q'}{P|Q \xrightarrow{\tau} P'|Q'} \text{ L-react}$$

$$\frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q} \text{ L-parallel L}$$

$$\frac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'} \text{ L-Par R}$$

α cannot be $a, \bar{a}$. τ, and $\notin \{a, \bar{a}\}$ ok

$$\frac{P \xrightarrow{\alpha} P' \qquad \alpha \notin \{a, \bar{a}\}}{\text{new } a\ P \xrightarrow{\alpha} \text{new } a\ P'} \text{ L-restriction}$$

Note: notation for substitution is not agreed upon.
$P_A[\vec{a} := \vec{b}]$ is another one

$$\frac{\{\vec{b}/\vec{a}\}\ P_A \xrightarrow{\alpha} P' \qquad A(\vec{a}) = P_A}{A\langle\vec{b}\rangle \xrightarrow{\alpha} P'} \text{ L-Identity}$$

Property:

If $P \xrightarrow{\alpha} P'$ and $P \equiv Q$ then $\exists Q'$.

$Q \xrightarrow{\alpha} Q'$ and $Q' \equiv P'$.

Proof: By induction on $P \xrightarrow{\alpha} P'$

Case L-Par L:

$P = P_1 | P_2 \qquad P_1 \xrightarrow{\alpha} P_1'$

$P' = P_1' | P_2$

Consider all the ways $P \equiv Q$

→ Subcase $Q = P_2 | P_1$

pick $Q' = P_2 | P_1'$

$Q \xrightarrow{\alpha} Q'$ by L-Parallel R

Aside: structural congruence

Def: $[\equiv]$

"Rewrite Rules"

1) α-conversion
2) reorder sums
3) $P|0 \equiv P$, $P|Q \equiv Q|P$,
$P|(Q|R) \equiv (P|Q)|R$
4) new $a\,(P|Q) \equiv$
$(\text{new } a\ Q)|P$ iff
$a \notin$ free variables (Q),
new $a\ 0 \equiv 0$,
new $a\ b\ P \equiv$ new $b\ a\ P$
5) $A\langle\vec{b}\rangle \equiv \{\vec{b}/\vec{a}\}\ P_A$
where $A(\vec{a}) = P_A$

"hole"

→ Subcase $Q = Q_1 \mid P_2$

$Q_1 \equiv P_1$

by induction hypothesis

$\exists Q'$ such that $Q_1 \xrightarrow{\alpha} Q_1' \equiv P_1'$

by L Par L, $Q \xrightarrow{\alpha} Q' \equiv P'$

$C ::= [\,] \mid \alpha \cdot C + M \mid C \mid P \mid \ldots$

↳ context

$C[P]$

$P \cong Q \Rightarrow C[P] \cong C[Q]$

basically this is the formal way of saying this

→ The LTS from today is equivalent to last time's system modulo labels.

<u>Theorem</u>

$P \to P'$ iff $P \xrightarrow{\tau} \equiv P'$

relational composition