Logarithms and Exponents

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17:16

Quiz

$\sigma_j = 1 \quad 2\tan^{-1}(2^{-(j+1)})$

$\sigma_j = -1 \quad -2\tan^{-1}(2^{-(j+1)})$

$\sigma_j = 0 \quad 0$

what if we add the following?

allow for a rotation by 0

rotate ↑ then ↓ to cancel out

$K = \prod_{j=0}^{n}\left(1+2^{-2(j+1)}\right)$

no, it doesn't support early rotations because we need to account for the x and y update due to the ↑↓ rotation of $\sigma_j = 0$

We can do this to allow redundancy in the representation,

So we can get away by not adding all of z to determine the sign, which allows us to use CSA's, and save on time.

Exponents

this method is a linear convergence algorithm with "Additive Normalization"

Try to compute $y = e^x$ we know x, want to calculate y

implemented as a lookup table

$x_{i+1} = x_i - \ln b_i$

$y_{i+1} = y_i \cdot b_i$

we start @ $x_0 = 0$, then $x_n = X$

$y_0 = 1$, and as we update x additively, we update y multiplicatively, and the final result, y_n, will be the y we want

sign, shift

choose $b_i \triangleq 1 + \sigma_i 2^{-i}$

thus,

$y_{i+1} = y_i + y_i \sigma_i 2^{-i}$

this gives us a range

$\sum \ln(1-2^{-i}) \le x_0 \le \sum \ln(1+2^{-i})$

if we do it till ∞, $-1.24 \le x_0 \le 1.56$

What about for logarithms?

$y = \ln x$ we know x, we want y

$x_{i+1} = x_i b_i$

$y_{i+1} = y_i - \ln b_i$

we want $x_n \to 1$, because

if $x_n = 1$, $x_0 \prod_{i=0}^{n} b_i = 1$

$\Rightarrow x_0 = \dfrac{1}{\prod_{i=0}^{n} b_i}$

$x_0 = x$

$$y_n = y_0 - \sum_{i=0}^{n} \ln b_i = y_0 + \ln\left(\frac{1}{\prod_{i=0}^{n} b_i}\right) = y_0 + \ln x$$

y_0 should be 0, to make sure it doesn't show up here

Sines and Cosines are typically calculated w/ software so we could look at math.c to look at the actual algorithms. OMG Quiz! Cawl! j/k