Reserve Reading

- The book “Iterative Computer Algorithms with Applications in Engineering” by A.M. Sait and H. Youssef, IEEE Publications (now Wiley) , 2000 is the primary source of materials for the material on the theory.
- Since I have exceeded already the allowed pages for xeroxing from this book, I cannot distribute more xeroxed pages from this book.
- However, the book is on reserve in Uris Library (under ORIE 5340 or CEE5290). You can borrow it for 3 hours during which time you can also xerox it if you wish for yourself.
- The relevant pages are: 18-26, 57-66 for random, greedy and simulated annealing.
- As discussed at the beginning of the course, you can also buy this paperback book online. The Amazon price is about $100 for a new copy.
    - Handout 10-24-11

SIMULATED ANNEALING (corrected)

Assume we have a scalar optimization problem with only 4 possible values for the decision variable. The cost of the 4 values is given inside the circle. The neighborhood is just adjacent points. What is the configuration graph for a simulated annealing search?

Basis for Configuration Graph Fig. 2.7

How do we work this out? We assume equal probability of picking any of the neighbors. So going from [1] there is a prob p_{1j} of 1/3 of picking 2 or 4 as neighbor or picking 1.

Transition Probability θ for 4 situations (General Expression)

$$\theta_{ij}(T) = \begin{cases} \dfrac{1}{|N(S_i)|} & \text{if } \Delta Cost_{ij} \le 0 \quad S_j \in N(S_i) \\ & \text{(downhill)} \\ \dfrac{1}{|N(S_i)|} e^{-\Delta Cost_{ij}/T} & \text{if } \Delta Cost_{ij} > 0, \; S_j \in N(S_i) \\ & \text{(uphill)} \\ 1 - \sum_{k \neq i} p_{ik} A_{ik}(T) & \text{if } i = j \quad S_j \in N(S_i) \\ & \text{(no move)} \\ 0 & S_j \notin N(S_i) \\ & \text{no "edge"} \end{cases}$$

This is the f(ci,cj,T)

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Example from ① to ③ $\in N(1)$ so S_j is in Neighborhood

$S(1) = 1$, $S(3) = 3$ so $\swarrow P_{22}$ $\swarrow P_{27}$ uphill move $N(2) = 3 - 1 = 2$

~~but as~~ In example neighborhood is defined so that the $S_i \in N(S_i)$

(so you could choose to stay at S_i even if it is worst solution.)

$\pi(t) = (\pi_1(t), \pi_2(t), \ldots, \pi_n(t))$ probability

state vector $\pi_i(t) =$ prob being in state i of

P (rowvector

matrix)

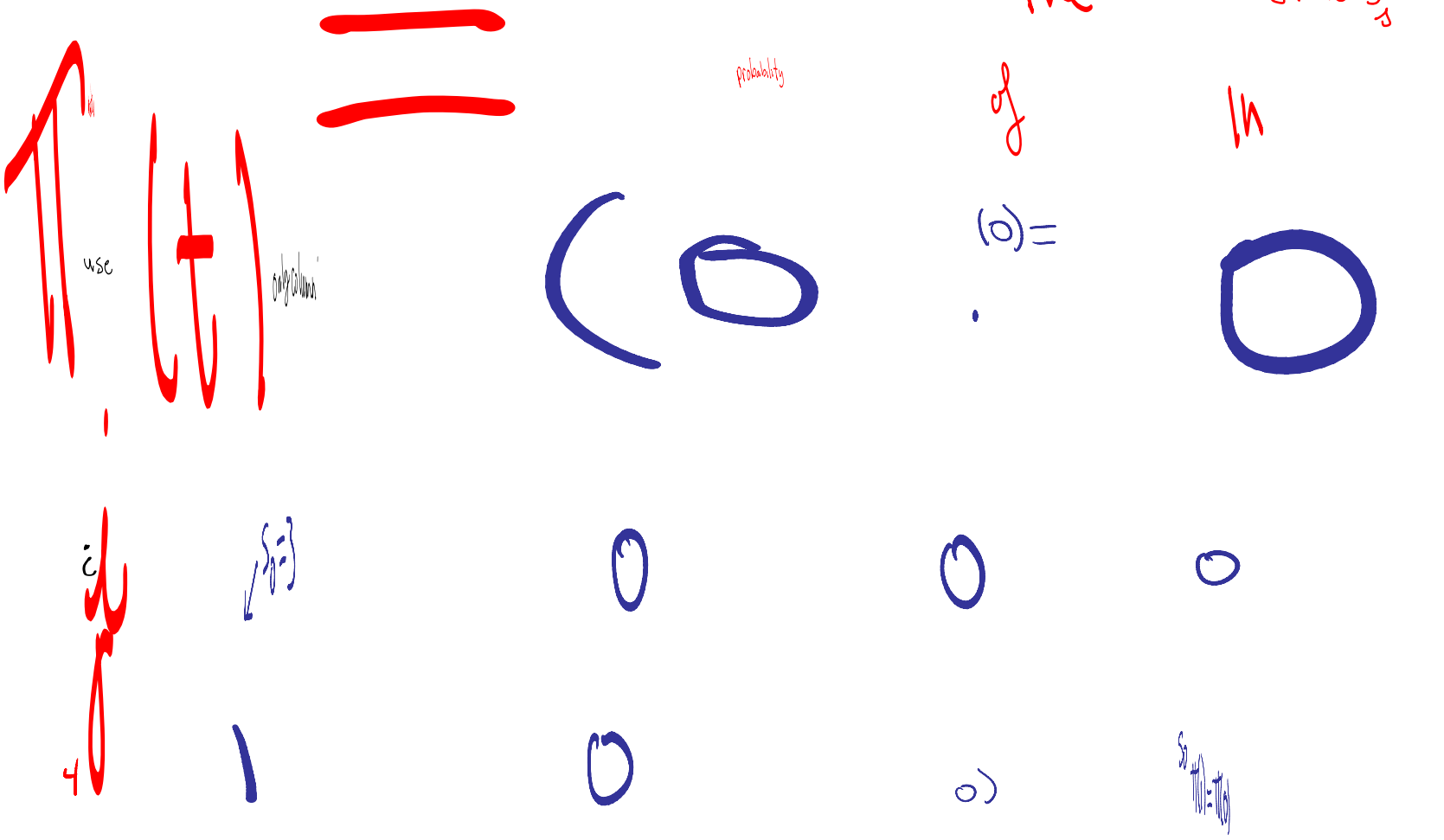
interation t π $P = [p_{ij}]$ $= \pi$

$(t+1)$ $= \pi(t)$

$\pi_i(t+1)$

Value of Θ for $\Theta_{ij} = p_{ij} A_{ij}$ for Fig. 2.7 Repeat Slide

Column number:



Everywhere there is an "e", there is an uphill move

Diagonal is prob of not moving

Example 2.3 and Fig. 2.7

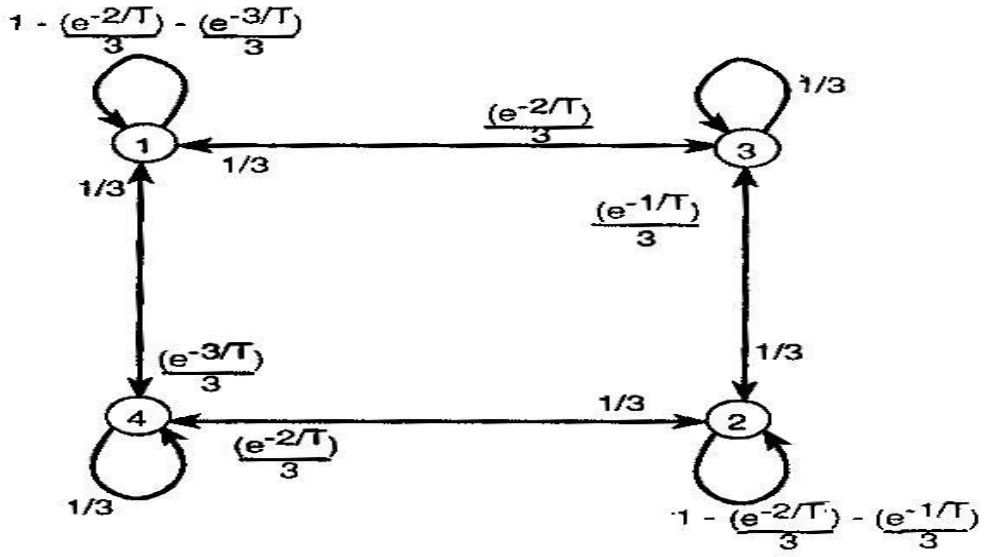


Figure 2.7: Configuration graph for Example 2.3.

Example 2.3 Consider the configuration graph corresponding to the Markov chain given in Figure 2.7. The cost of the four states is as follows: $Cost_1 = 1$, $Cost_2 = 2$, $Cost_3 = 3$, and $Cost_4 = 4$, where $Cost_n$ represents the cost of state S_n. The acceptance function is given by the Metropolis criterion $f(Cost_i, Cost_j, T) = e^{-\frac{\Delta Cost_{ij}}{T}}$. Verify that the transition matrix for the Markov chain is as given below and obtain the stationary distribution.

What do we do next?

- So now that we have the characteristics of the configuration graph, we would like to know if it will converge to the best solution no matter which is your guess for the initial value So.
- Now we will discuss the convergence analysis and then apply it to this specific problem.

Theoretical Results indicate the convergent π* is obtained by Double limit analysis

To find the final solution of the eventual SA search, we do the following:

1. Holding *T* constant, let *t* (number of iterations) go in the limit to infinity)
2. Given the limit in 1 (which is a function of *T*), let *T* go in the limit to 0. (T is the temperature parameter)

Theoretical Results indicate π* is obtained by Double limit analysis

this can be expressed as:

Overall Theory for Stationary Distribution of Simulated Annealing with Fixed T

- It has been proven that in general (i.e. for all ergodic chains) that

is the solution P

- $\pi_i^{(T)} =$ which since to the $= \pi(0) P^t = \pi^*$

π^*

- following $\sum_{S_j \in S} \exp(-\text{cost}_i / T)$ (a normalizing factor)

- Note that the value of the parameter T is fixed.

Theoretical Results indicate π^* is obtained by Double limit analysis

To find the final solution of the eventual SA search, we do the following:

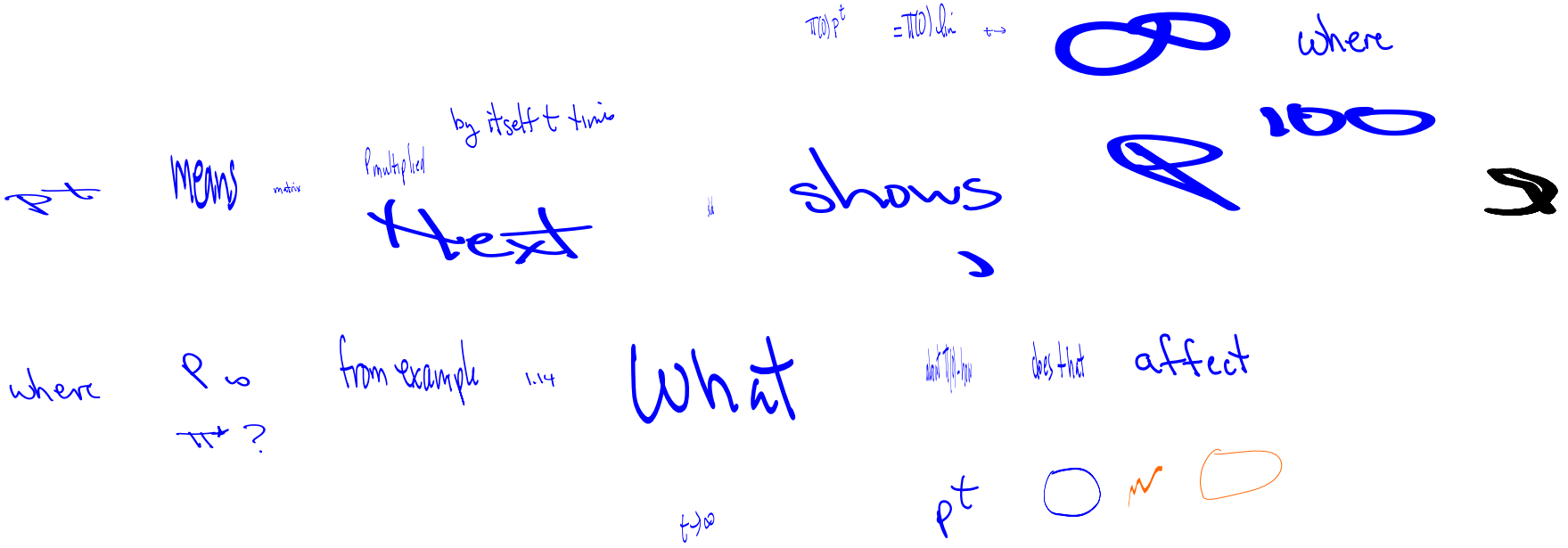
1.) Holding *T* constant, let *t* (number of iterations) go in the limit to infinity)

2.) Given the limit in 1.) (which is a function of *T*), let *T* go in the limit to 0.

(*T* is the temperature parameter)

SA GENERAL THEORETICAL RESULTS
Lim t goes to infinity and then Lim T goes to zero
(incorporates equations from previous slide)

- From Ergodic Markov Chain Theory for Simulated Annealing we know that



Where Ω_o is the set of optimal solutions so $Cost_j$ is equal for all members of Ω_o.

If the unique global optimium is at k, what is limit of π_k (T) as T goes to 0?

Solving Fig. 2.7/Example 2.3

- If we solve the equation $\pi^* = \pi^* \Theta$ (e.g. using Matlab), we get:

Limit as T goes to 0 for Example 2.3/Fig. 2.7 and Consistency with General SA Convergence Results

- N(T) can be written as:

Where:

- So then

Hence our convergence analysis for Ex. 2.3 is consistent with General Results given in earlier slide since |Ω| =1 for this example.

What Can We Conclude From This Example?

- If you use 1000 iterations of SA, will you find the best solution?

- What does the theory tell you?

SA with T=0 is Greedy Search

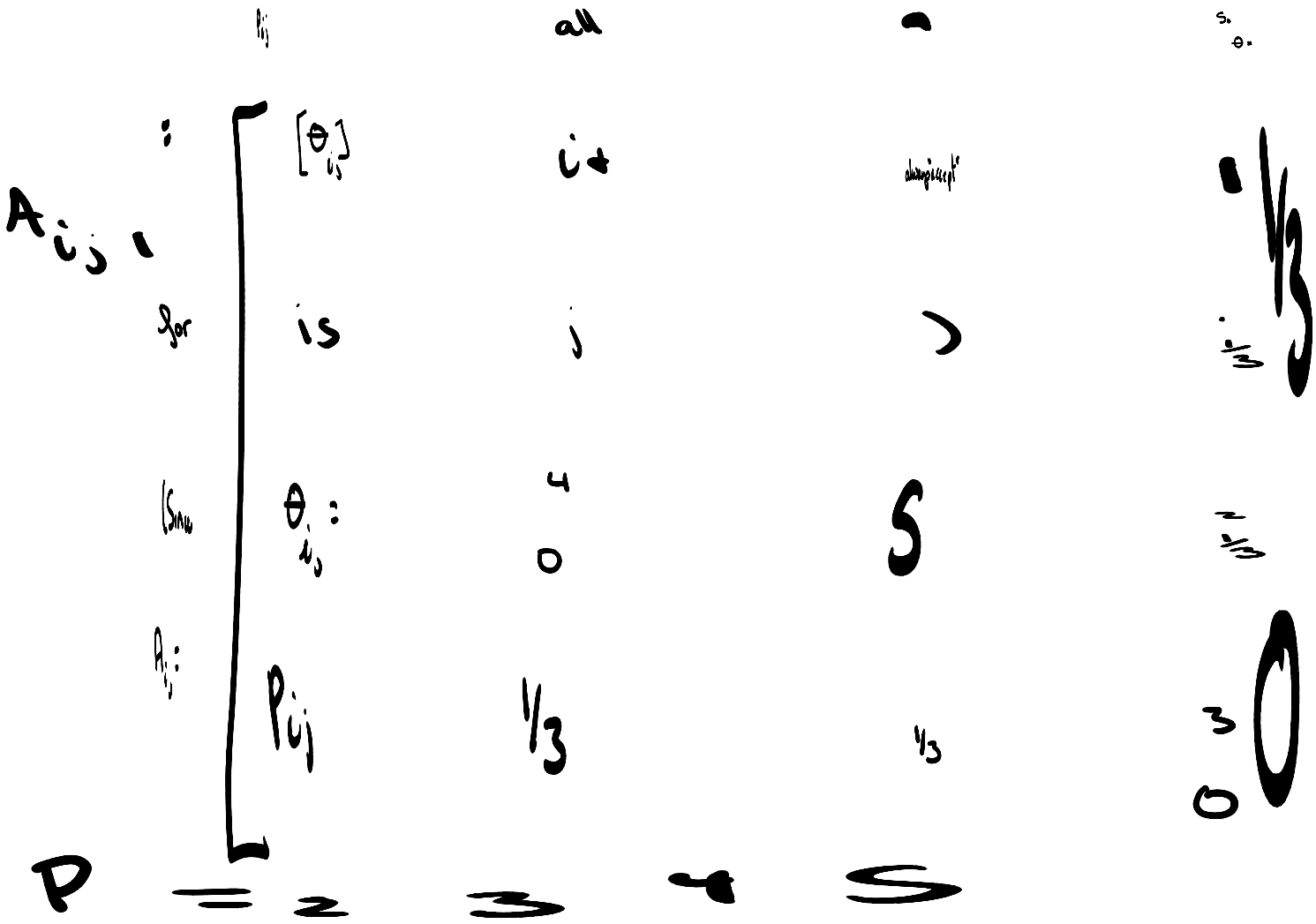
- We might ask-- why bother with SA since we let T approach 0 in the limit in the convergence?

- Why not just let T=0 to begin with?

- The next slide shows the transition matrix for example 2.3/Figure 2.7 if we set T=0 (before taking limit t goes to infinity)

What does the transition matrix for Example 2.3 Fig 5.7 look like if we set T=0 and $e^{-K/T}$ to 0

(Is this a global optimization algorithm with T=0?)



Consider the case where T=0 and t=0

Case 1: What happens if you start in state 1 and T=0? Then

Case 2: What happens if you start in state 2 and T=0? Then

- What does this tell you? Is the transition matrix for the case T=0 for an ergodic chain?